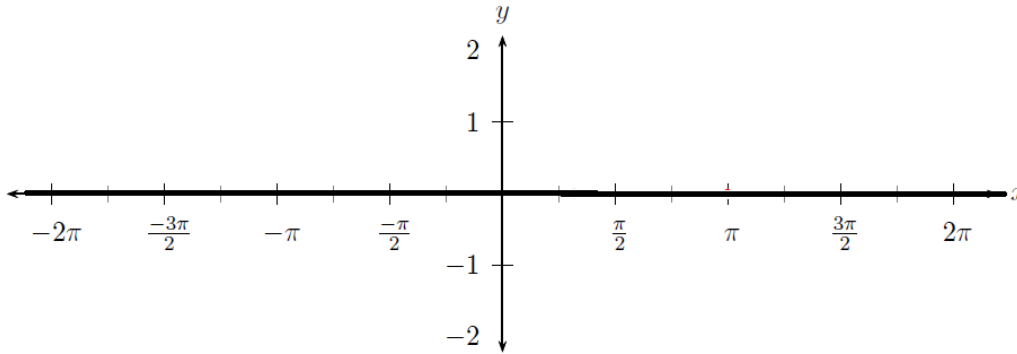


What you'll Learn About

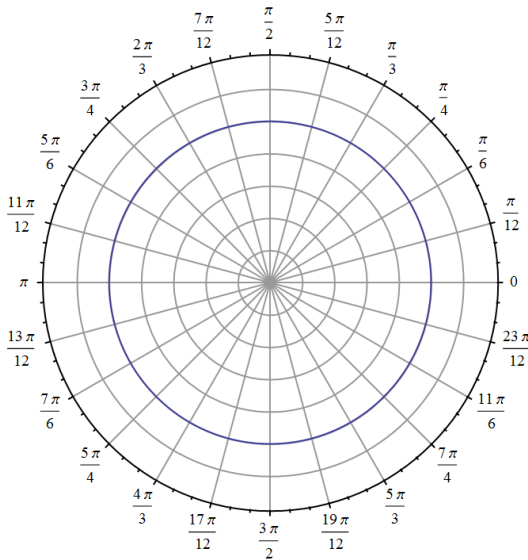
- Polar Curves and Parametric Curves/Symmetry/Analyzing Polar Graphs
- Rose Curves/Limacon Curves/Other Polar Curves

A) Graph the following:  $y = 4\sin x$

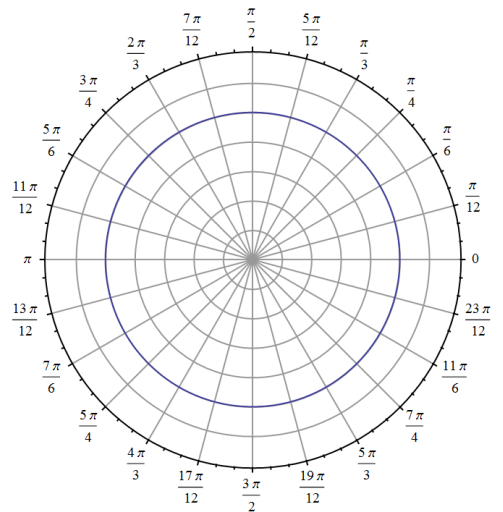


Graph the following polar curve:

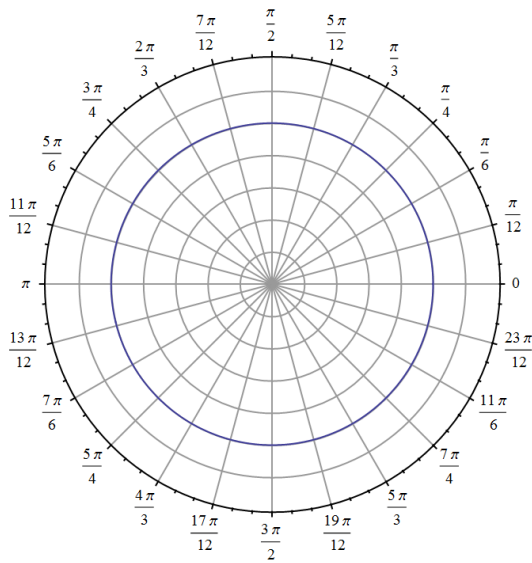
A)  $r = 4\sin \theta$



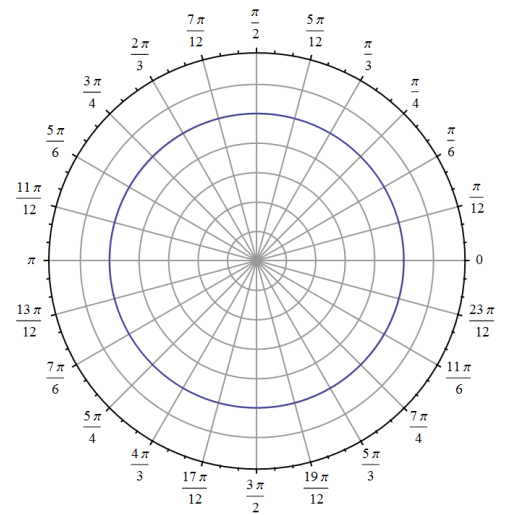
B)  $r = -2\sin \theta$



Graph the following polar curve:  $r = 4\cos\theta$



Graph the following polar curve:  $r = -2\cos\theta$

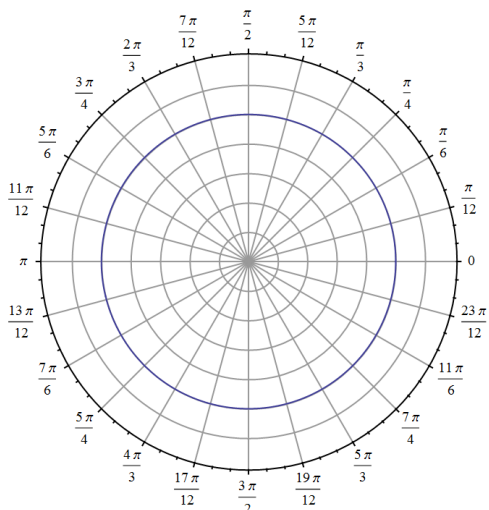


### Summary of Polar Circles

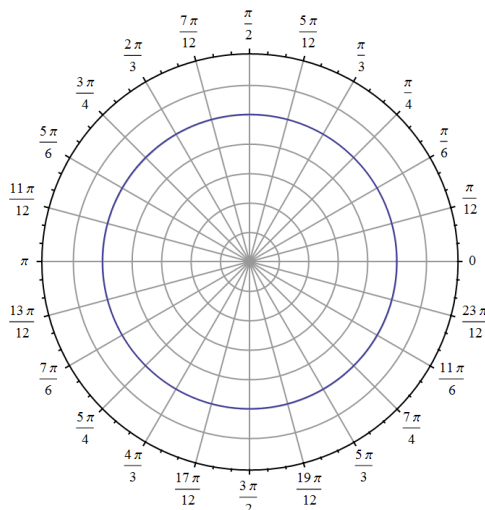
- Circles always move counterclockwise
- The circle completes itself from 0 to  $\pi$ .
- Going from 0 to  $2\pi$  would retrace the original circle
- The number in front of sine is  $r$  (the diameter of the circle)
- Equations with positive sine start at the origin and then move counterclockwise up
- Equations with negative sine start at the origin and then move counterclockwise down
- Equations with positive cosine start at the radius on the positive side of the pole and then move counterclockwise up and back toward the origin
- Equations with negative cosine start at the radius on the negative side of the pole and then move counterclockwise down and back toward the origin

Graph the following polar curve:

$$r = 2 - 2\cos\theta$$

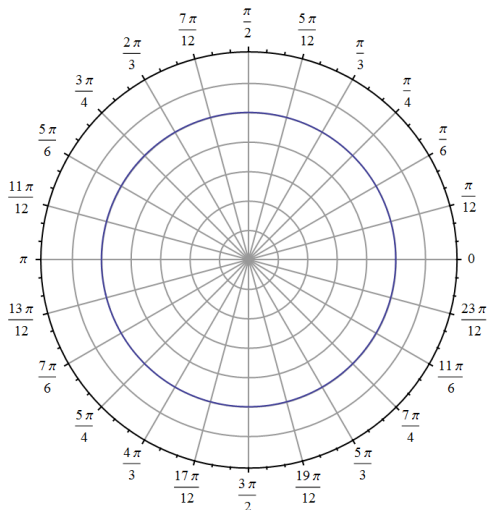


$$r = 3 - 3\cos\theta$$

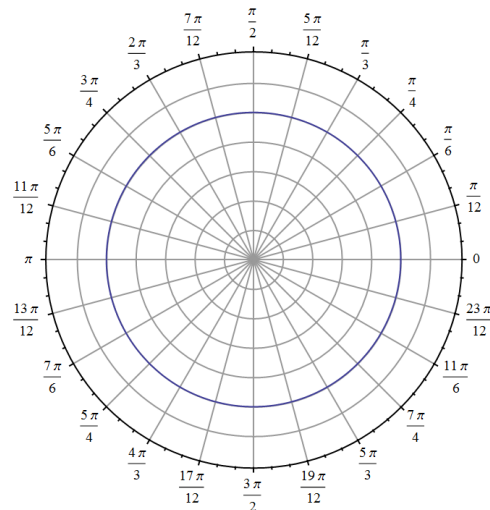


Graph the following polar curve:

$$r = -2 + 2\cos\theta$$

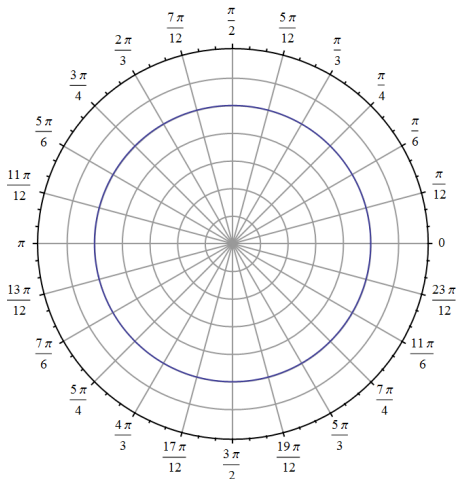


$$r = -3 + 3\cos\theta$$

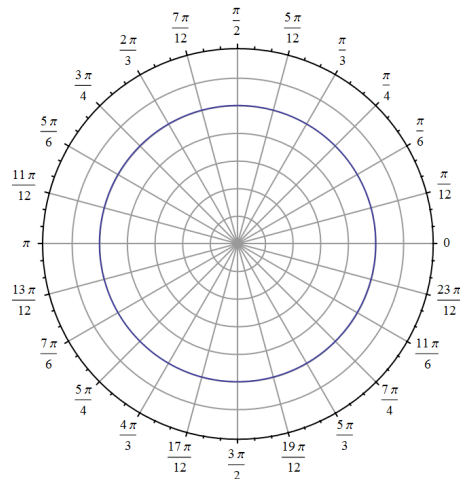


Graph the following polar curve:

$$r = 2 + 2\cos\theta$$

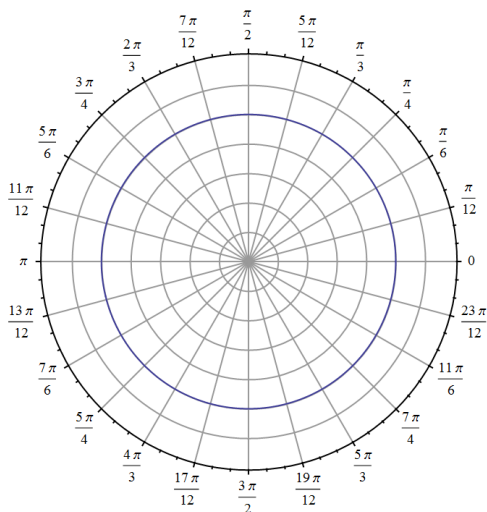


$$r = 3 + 3\cos\theta$$

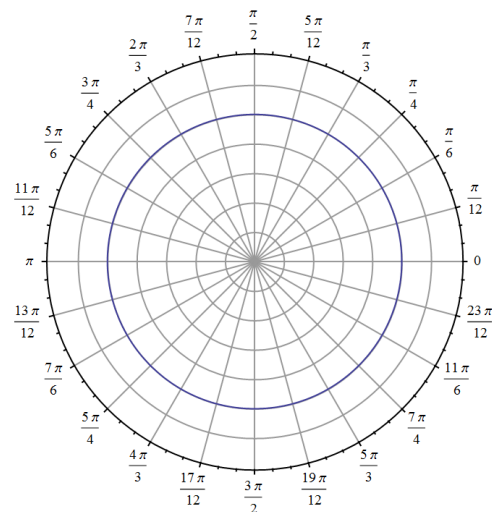


Graph the following polar curve:

$$r = 2 - 2\sin\theta$$

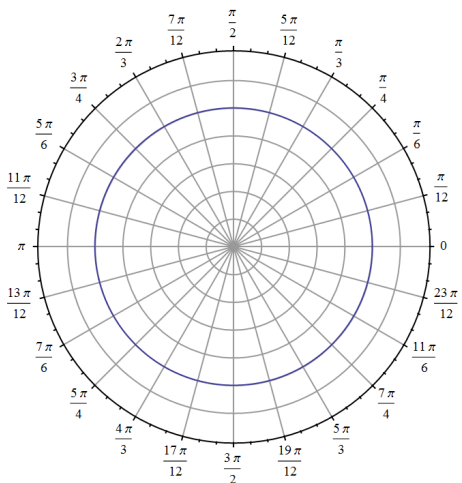


$$r = 3 - 3\sin\theta$$

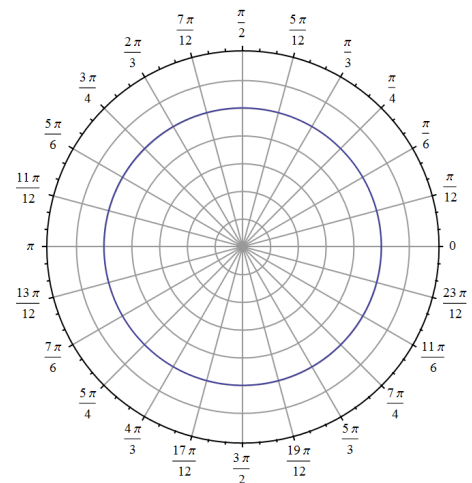


Graph the following polar curve:

$$r = -2 + 2\sin\theta$$



$$r = -3 + 3\sin\theta$$

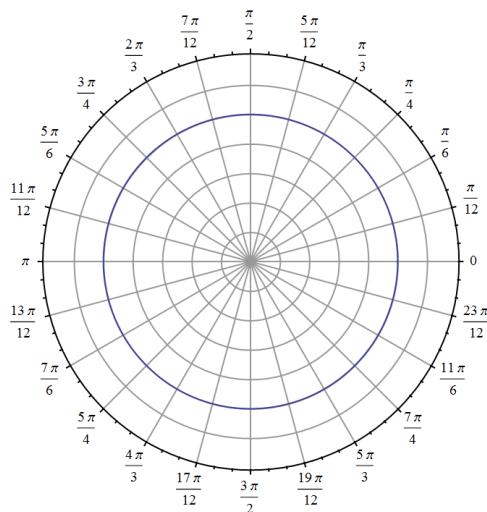


Summary of the Cardioid:  $r = \pm a \pm b \cos\theta$

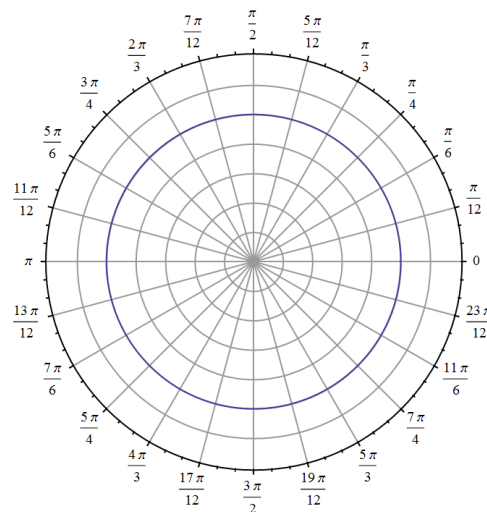
- For a polar equation to be considered a cardioid, the values of  $|a|$  and  $|b|$  must be the same.
- Plug in  $\theta = 0$ . This will give you the value of  $r$  and where you will start moving counterclockwise.
- The value of  $a$  will tell you where the curve is at on the  $y$ -axis (when  $\theta = \frac{\pi}{2}$  and  $\theta = \frac{3\pi}{2}$ )
- To complete the entire shape  $0 \leq \theta \leq 2\pi$
- The value of  $|a| + |b|$  will tell you how far out on the  $x$ -axis the curve is
- If  $b$  is negative the curve will be on the left side of the pole
- If  $b$  is positive the curve will be on the right side of the pole

Graph the following polar curve:

$$r = 2 - 3\cos\theta$$

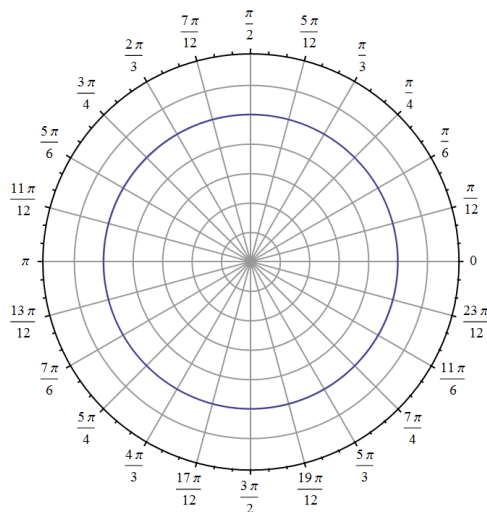


$$r = 2 - 4\cos\theta$$

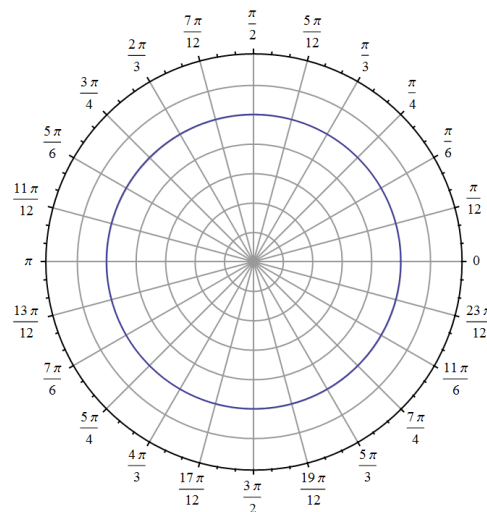


Graph the following polar curve:

$$r = 2 + 3\sin\theta$$

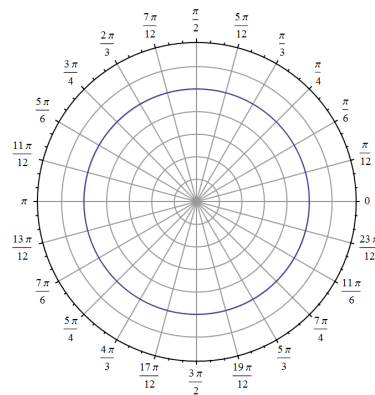


$$r = 2 - 3\sin\theta$$

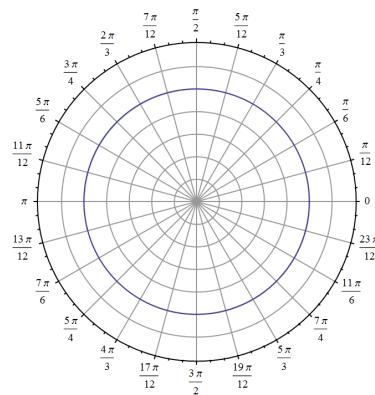


Graph the following polar curve:

$$r = 2 + 3\sin\theta$$



$$r = 2 + 4\sin\theta$$



Summary of the Limacon:  $r = \pm a \pm b \cos \theta$

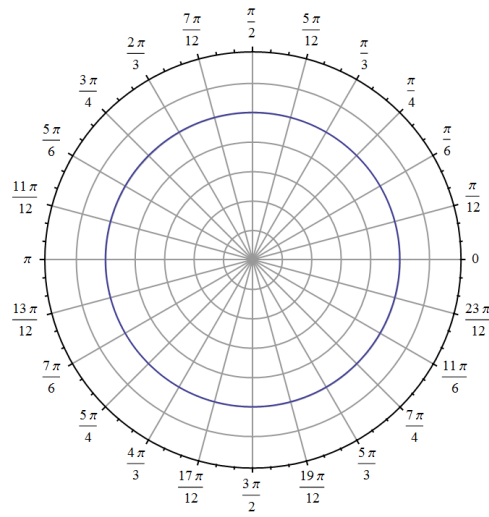
- For a polar equation to be considered a Limacon with a loop, the value  $|a|$  must be smaller than  $|b|$ .
- Plug in  $\theta = 0$ . This will give you the value of  $r$  and where you will start the curve moving counterclockwise back to the pole.
- The value of  $a$  will tell you where the curve is at on the  $y$ -axis (when  $\theta = \frac{\pi}{2}$  and  $\theta = \frac{3\pi}{2}$ )
- To complete the entire shape  $0 \leq \theta \leq 2\pi$ .
- The value of  $|a| + |b|$  will tell you how far out on the  $x$ -axis the curve is
- The value of  $|a| - |b|$  will tell you how far out on the  $x$ -axis the loop is
- If  $b$  is negative the curve and the loop will be on the left side of the pole
- If  $b$  is positive the curve and the loop will be on the right side of the pole

Summary of the Limacon:  $r = \pm a \pm b \sin \theta$

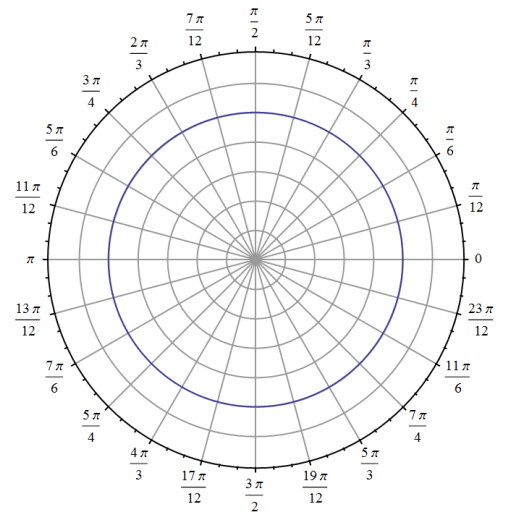
- For a polar equation to be considered a Limacon with a loop, the value  $|a|$  must be smaller than  $|b|$ .
- Plug in  $\theta = 0$ . This will give you the value of  $r$  and where you will start the curve moving counterclockwise back to the pole.
- The value of  $a$  will tell you where the curve is at on the  $x$ -axis (when  $\theta = 0$  and  $\theta = \pi$ )
- To complete the entire shape  $0 \leq \theta \leq 2\pi$ .
- The value of  $|a| + |b|$  will tell you how far out on the  $y$ -axis the curve is
- The value of  $|a| - |b|$  will tell you how far out on the  $y$ -axis the loop is
- If  $b$  is negative the curve and the loop will be below the pole
- If  $b$  is positive the curve and the loop will be above the pole

Graph the following polar curve:

$$r = 3 + 2\sin\theta$$

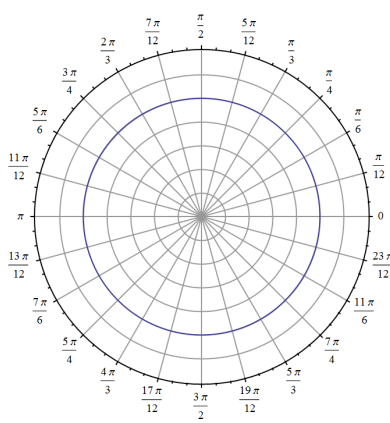


$$r = 3 - 2\cos\theta$$

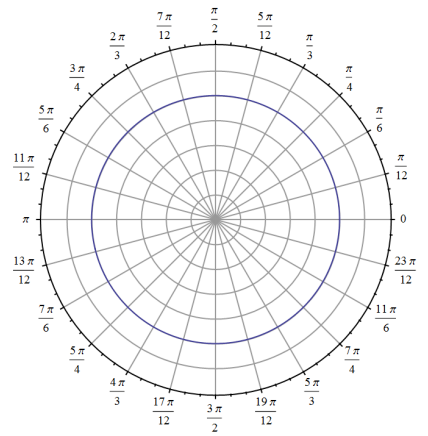


Graph the following polar curve:

$$r = -3 - 2\sin\theta$$



$$r = -3 + 2\cos\theta$$



Summary of the Dimpled Limacon:  $r = \pm a \pm b \cos \theta$

- For a polar equation to be considered a Dimpled Limacon, the value  $|a|$  must be larger than  $|b|$ .
- Plug in  $\theta = 0$ . This will give you the value of  $r$  and where you will start the curve moving counterclockwise.
- There will be no value at the pole.
- The value of  $a$  will tell you where the curve is at on the  $y$ -axis (when  $\theta = \frac{\pi}{2}$  and  $\theta = \frac{3\pi}{2}$ )
- To complete the entire shape  $0 \leq \theta \leq 2\pi$ .
- The value of  $|a| + |b|$  will tell you how far out to the right on the  $x$ -axis the curve is if cosine is positive
- The value of  $|a| + |b|$  will tell you how far out to the left on the  $x$ -axis the curve is if cosine is negative
- The value of  $|a| - |b|$  will tell you how far out on the left of the  $x$ -axis the curve is if cosine is positive
- The value of  $|a| - |b|$  will tell you how far out on the right of the  $x$ -axis the curve is if cosine is negative

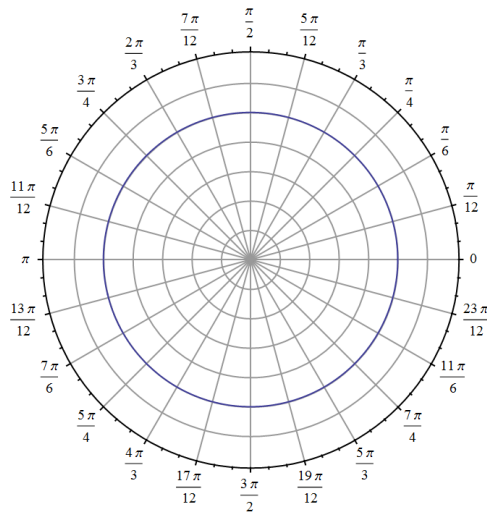
Summary of the Limacon:  $r = \pm a \pm b \sin \theta$

- For a polar equation to be considered a Dimpled Limacon, the value  $|a|$  must be larger than  $|b|$ .
- Plug in  $\theta = 0$ . This will give you the value of  $r$  and where you will start the curve moving counterclockwise.
- There will be no value at the pole.
- The value of  $a$  will tell you where the curve is at on the  $x$ -axis (when  $\theta = 0$  and  $\theta = \pi$ )
- To complete the entire shape  $0 \leq \theta \leq 2\pi$ .
- The value of  $|a| + |b|$  will tell you how far up on the  $y$ -axis the curve is if sine is positive
- The value of  $|a| + |b|$  will tell you how far up on the  $y$ -axis the curve is if sine is negative
- The value of  $|a| - |b|$  will tell you how far down on the  $y$ -axis the curve is if sine is positive
- The value of  $|a| - |b|$  will tell you how far up on the  $y$ -axis the curve is if sine is negative

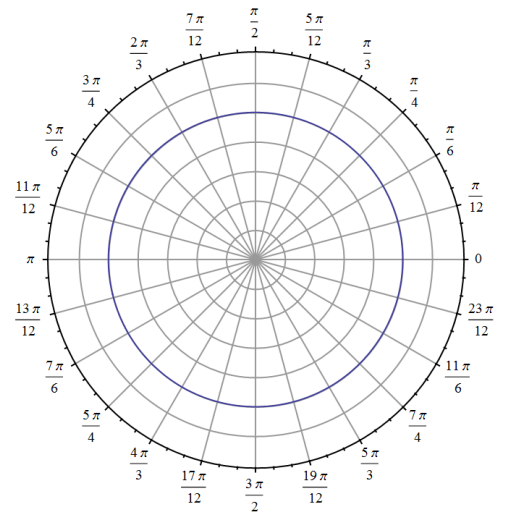


Graph the following polar curve:

$$r = 2\cos 2\theta$$

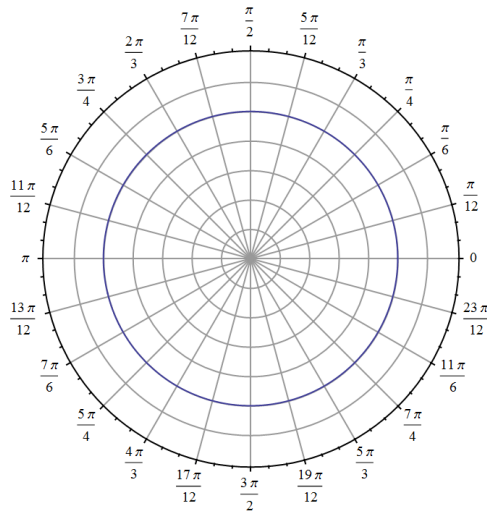


$$r = 2\cos 3\theta$$

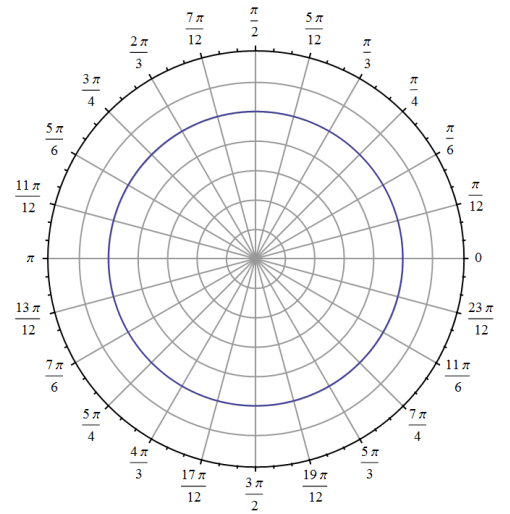


Graph the following polar curve:

$$r = 2\sin 2\theta$$



$$r = 2\sin 3\theta$$



Summary of the Rose:  $r = \pm a \cos b\theta$

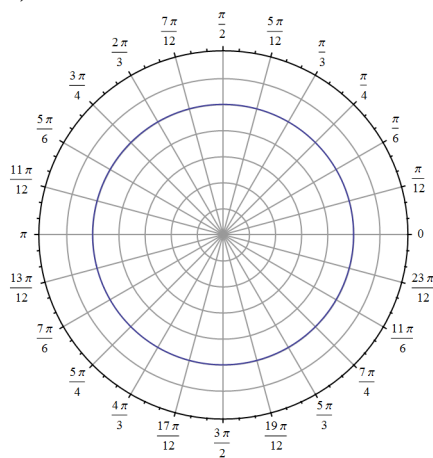
- For a polar equation to be considered a Rose, the value  $b$  must be greater than 1
- Plug in  $\theta = 0$ . This will give you the value of  $r$  and where you will start the the curve moving counterclockwise back to the pole.
- The value of  $a$  will tell you how far the furthest point away from the pole is (These occur at the tips of the rose petals)
- Divide 360 by the number of petals. Keep adding this number until you get back to 360. This will be where the tips of the petals are.
- If  $b$  is an odd number, that is the number of rose petals.
- If  $b$  is an even number, there are  $2b$  number of rose petals.
- If  $b$  is negative the curve will be the same as if  $b$  is positive because cosine is an even function.
- If  $a$  is a negative number, the rose petal starts on the left side of the pole
- If  $a$  is a positive number, the rose petal starts on the right side of the pole

Summary of the Rose:  $r = \pm a \sin b\theta$

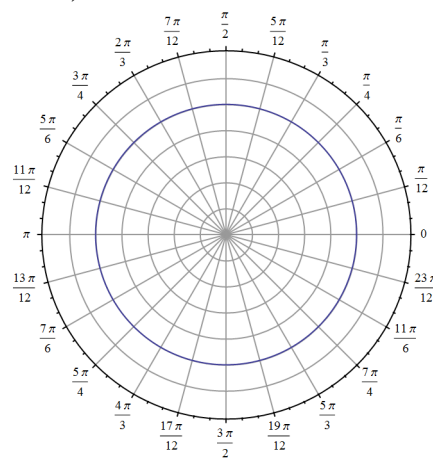
- For a polar equation to be considered a Rose, the value  $b$  must be greater than 1
- Plug in  $\theta = 0$ . This will give you the value of  $r$  and where you will start the the curve moving counterclockwise out from the pole.
- The value of  $a$  will tell you how far the furthest point away from the pole is (These occur at the tips of the rose petals)
- Divide 90 by  $b$ . This will be where the end of the first petal is.
- Divide 360 by the number of petals. Keep adding this number to your first tip until you get back to 360. This will be the angles where the tips of the petals are.
- If  $b$  is an odd number, that is the number of rose petals.
- If  $b$  is an even number, there are  $2b$  number of rose petals.
- If  $a$  and  $b$  are both positive or both negative the rose curve will open into the first quadrant.
- If  $a$  or  $b$  is negative the rose curve will open into the third quadrant, because sine is an odd function

## Lemiscate

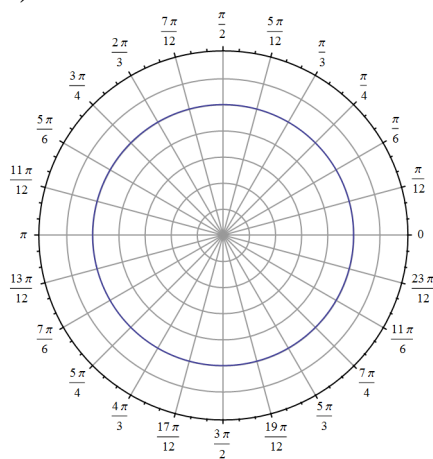
A)  $r^2 = 4\cos\theta$



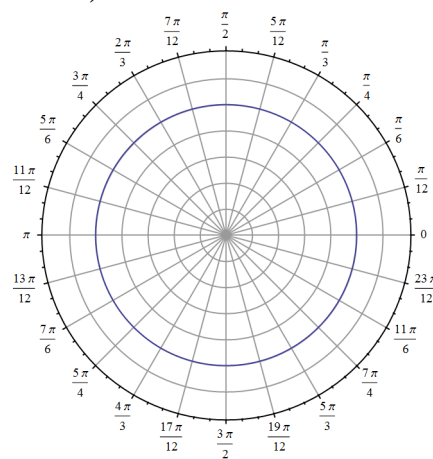
B)  $r^2 = 4\sin\theta$



A)  $r^2 = 4\cos 2\theta$



B)  $r^2 = 4\sin 2\theta$



Summary of Lemniscate:  $r^2 = a \cos b\theta$

- Figure 8 will be on the x-axis if  $b = 1$  or  $2$

Summary of Lemniscate:  $r^2 = a \sin b\theta$

- Figure 8 will be on the y-axis if  $b = 1$
- Figure 8 will be in quadrant 1 and 3 if  $b = 2$