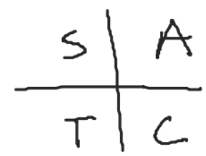
In Exercises 43–48, evaluate without using a calculator.



43. Find $\sin\theta$ and $\tan\theta$ if $\cos\theta = \dfrac{2}{3}$ and $\cot\theta > 0$.

44. Find $\cos\theta$ and $\cot\theta$ if $\sin\theta = \dfrac{1}{4}$ and $\tan\theta < 0$.

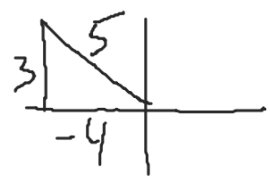
45. Find $\tan\theta$ and $\sec\theta$ if $\sin\theta = -\dfrac{2}{5}$ and $\cos\theta > 0$.

46. Find $\sin\theta$ and $\cos\theta$ if $\cot\theta = \dfrac{3}{7}$ and $\sec\theta < 0$.

$\cot\theta = \dfrac{x}{y}$ $\sin\theta = \dfrac{-7}{\sqrt{58}}$, $\cos\theta = \dfrac{-3}{\sqrt{58}}$

47. Find $\sec\theta$ and $\csc\theta$ if $\cot\theta = -\dfrac{4}{3}$ and $\cos\theta < 0$.

$\sec\theta = -\dfrac{5}{4}$ $\csc\theta = \dfrac{5}{3}$



48. Find $\csc\theta$ and $\cot\theta$ if $\tan\theta = -\dfrac{4}{3}$ and $\sin\theta > 0$.

Using Reference Angles to Find Trigonometric Functions

Use reference angles to find all six trigonometric functions of $-\frac{5\pi}{6}$.

Use reference angles to find all six trigonometric functions of $-\frac{7\pi}{4}$.

Even and Odd Functions

An even function is one in which $f(-x) = f(x)$.

An odd function is one in which $f(-x) = -f(x)$.

Even

$f(1) = f(-1)$

$f(x) = x^4$

$f(1) = (1)^4 = 1$

$f(-1) = (-1)^4 = 1$

$\cos(45^\circ) = \frac{\sqrt{2}}{2}$

$\cos(-45^\circ) = \frac{\sqrt{2}}{2}$

Odd

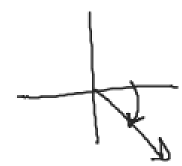
$f(1) = -f(-1)$

$f(x) = x^5$

$f(1) = 1^5 = 1$ $\quad f(-1) = (-1)^5 = -1$

$\sin(45^\circ) = \frac{\sqrt{2}}{2}$

$\sin(-45^\circ) = \frac{-\sqrt{2}}{2}$



Even	Odd
cos	sin/csc
sec	tan/cot

$\tan(45^\circ) = 1$

$\tan(-45^\circ) = -1$

Even

$\cos t = \cos(-t)$

$\sec t = \sec(-t)$

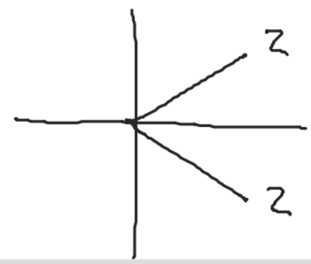
odd

$\sin(t) = -\sin(-t)$

Using Even and Odd Properties of Trigonometric Functions

If the secant of angle t is 2, what is the secant of $-t$?

$\sec t = 2$ $\sec(-t) = 2$



44-48

If the cotangent of angle t is $\sqrt{3}$, what is the cotangent of $-t$?

$\cot t = \sqrt{3}$ $\cot(-t) = -\sqrt{3}$

FUNDAMENTAL IDENTITIES

We can derive some useful identities from the six trigonometric functions. The other four trigonometric functions can be related back to the sine and cosine functions using these basic relationships:

$\cos t = x$

$$\tan t = \frac{\sin\ t}{\cos\ t}$$

$\sin t = y$

$$\sec\ t = \frac{1}{\cos\ t}$$

$\sec t = \frac{1}{x} = \frac{1}{\cos t}$

$\tan t = \frac{y}{x}$

$$\csc\ t = \frac{1}{\sin\ t}$$

$\csc t = \frac{1}{y} = \frac{1}{\sin t}$

$= \frac{\sin t}{\cos t}$

$$\cot t = \frac{1}{\tan t} = \frac{\cos t}{\sin t}$$

$= \frac{1}{\tan t} = \frac{\cos t}{\sin t}$

Using Identities to Evaluate Trigonometric Functions

a. Given $\sin(45^\circ) = \frac{\sqrt{2}}{2}$, $\cos(45^\circ) = \frac{\sqrt{2}}{2}$, evaluate $\tan(45^\circ)$.

b. Given $\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$, $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$, evaluate $\sec\left(\frac{5\pi}{6}\right)$.

$$\tan 45^\circ = \frac{\sin 45^\circ}{\cos 45^\circ} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

$$\sec \frac{5\pi}{6} = \frac{1}{\cos \frac{5\pi}{6}} = \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$$

Evaluate $\csc\left(\frac{7\pi}{6}\right)$.

$$\csc\frac{7\pi}{6} = \frac{1}{\sin\frac{7\pi}{6}} = \frac{1}{-\frac{1}{2}} = -2$$

Using Identities to Simplify Trigonometric Expressions

Simplify $\frac{\sec\ t}{\tan\ t}$.

$$\frac{\sec t}{\tan t} = \frac{\frac{1}{\cos t}}{\frac{\sin t}{\cos t}} \quad \frac{1}{\cancel{\cos t}} \cdot \frac{\cancel{\cos t}}{\sin t}$$

$$\frac{1}{\sin t}$$

Simplify $(\tan\ t)(\cos\ t)$.

$$\left(\frac{\sin t}{\cancel{\cos t}}\right)\left(\cancel{\cos t}\right) = \sin t$$

Alternate Forms of the Pythagorean Identity

$$1 + \tan^2 t = \sec^2 t$$

$$\cot^2 t + 1 = \csc^2 t$$

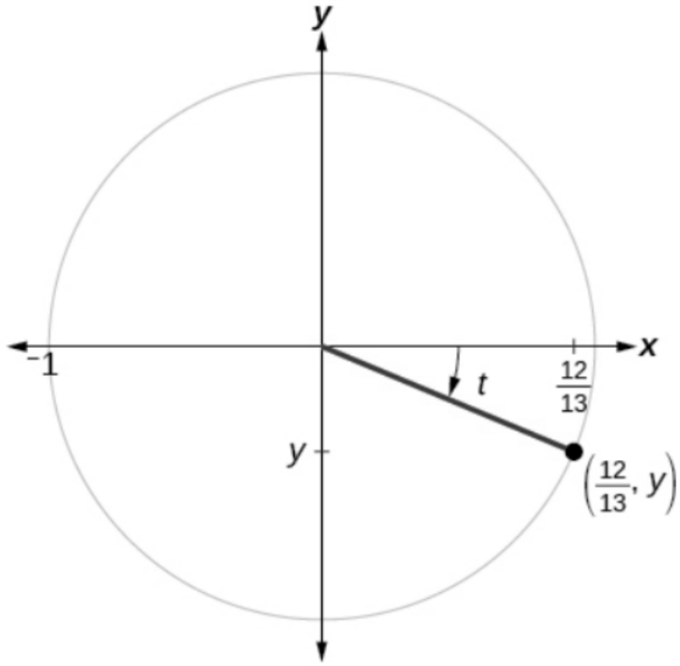
$$\frac{\cos^2 t}{\cos^2 t} + \frac{\sin^2 t}{\cos^2 t} = \frac{1}{\cos^2 t}$$

$$1 + \tan^2 t = \sec^2 t$$

$$\frac{\cos^2 t}{\sin^2 t} + \frac{\sin^2 t}{\sin^2 t} = \frac{1}{\sin^2 t}$$

$$\cot^2 t + 1 + \csc^2 t$$

If $\cos(t) = \frac{12}{13}$ and t is in quadrant IV, as shown in Figure 8, find the values of the other five trigonometric functions.



$$\left(\frac{12}{13}\right)^2 + \sin^2 t) = 1^2$$

$$\frac{144}{169} + \sin^2 t = \frac{169}{169}$$

$$\sin^2 t = \frac{25}{169}$$

$$\sin t = -\frac{5}{13}$$

$\sin\theta = \frac{-5}{13}$ $\csc\theta = -\frac{13}{5}$

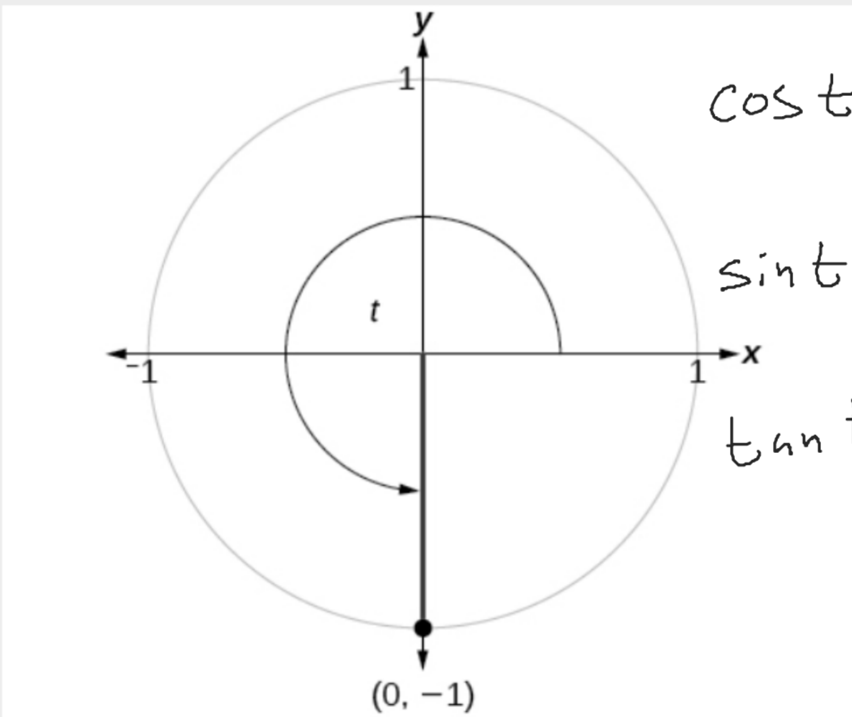
$\cos\theta = \frac{12}{13}$ $\sec\theta = \frac{13}{12}$

$\tan t = \dfrac{\frac{-5}{13}}{\frac{12}{13}}$ $\cot(t) = -\frac{12}{5}$

$= -\frac{5}{12}$

If $\sec(t) = -\frac{17}{8}$ and $0 < t < \pi$, find the values of the other five functions.

Find the values of the six trigonometric functions of angle t based on Figure 10.



$\cos t = 0$ $\sec\theta \to$

$\sin t = -1$ $\csc(t)$

$\tan t = \frac{-1}{0}$ $\cot$

$= \text{undefined}$

Evaluating the Cosecant Using Technology

Evaluate the cosecant of $\frac{5\pi}{7}$.

Evaluate the cotangent of $-\frac{\pi}{8}$.