Examples 4.7

Choosing a Mathematical Model

Does a linear, exponential, logarithmic, or logistic model best fit the values listed in Table 1? Find the model, and use a graph to check your choice.

x	1	2	3	4	5	6	7	8	9
y	0	1.386	2.197	2.773	3.219	3.584	3.892	4.159	4.394

Table 1

TRY IT #6

Does a linear, exponential, or logarithmic model best fit the data in Table 2? Find the model.

x	1	2	3	4	5	6	7	8	9
y	3.297	5.437	8.963	14.778	24.365	40.172	66.231	109.196	180.034

Table 2

For the following exercises, use this scenario: A tumor is injected with 0.5 grams of Iodine-125, which has a decay rate of 1.15% per day.

31. To the nearest day, how long will it take for half of the Iodine-125 to decay?

32. Write an exponential model representing the amount of Iodine-125 remaining in the tumor after t days. Then use the formula to find the amount of Iodine-125 that would remain in the tumor after 60 days. Round to the nearest tenth of a gram.

t

31) $y = ab^x$

$a = .5$ grams

$b = .9885$

$y = .5(.9885)^t$

$\frac{.25}{.5} = \frac{.5(.9885)^t}{.5}$

$.5 = .9885^t$

$\frac{\ln(.5)}{\ln(.9885)} = \frac{t\ln(.9885)}{\ln(.9885)}$

$t \approx 60$ days

$y = a(1-r)^x$

$(1 - 0.0115)$

$y = .5(.9885)^t$

$= .5(.9885)^{60}$

$\approx .2$ grams

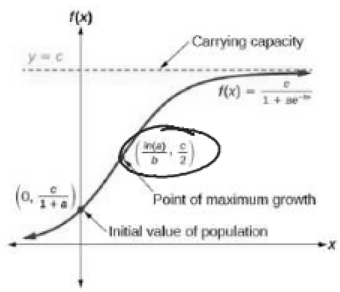
LOGISTIC GROWTH

The logistic growth model is

$$f(x) = \frac{c}{1+ae^{-bx}}$$ → maxium value

where

- $\frac{c}{1+a}$ is the initial value → $X=0$
- c is the *carrying capacity*, or *limiting value*
- b is a constant determined by the rate of growth.



An influenza epidemic spreads through a population rapidly, at a rate that depends on two factors: The more people who have the flu, the more rapidly it spreads, and also the more uninfected people there are, the more rapidly it spreads. These two factors make the logistic model a good one to study the spread of communicable diseases. And, clearly, there is a maximum value for the number of people infected: the entire population.

$$f(x) = \frac{1000}{1+999e^{-0.6030x}}$$ → max

$\frac{1000}{1+999} = \frac{1000}{1000} = 1$

1) What is the maximum amount of people who could get the flu? 1000

2) What is the initial number of people who had the flu? $x=0$ 1

3) Determine the number of flu cases there will be when the flu is spreading at the fastest rate?

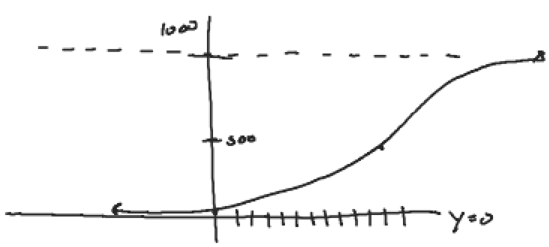
4) Sketch a graph of the function.

5) How many people will be infected with the flu after 8 days? 111 people

6) How long will it take for the flu to reach 400 cases?

$$3)\left(\frac{\ln(a)}{b}, \frac{c}{2}\right) = \left(\frac{\ln(999)}{.6030}, \frac{1000}{2}\right)$$

$(11.45, 500)$



$$(400) = \left(\frac{1000}{1+999e^{-.6030x}}\right)$$

$$\left(1+999e^{-.6030x}\right)(400) = 1000$$

$$2.5 = 1 + 999e^{-.6030x}$$

$$-1 \quad -1$$

$$\frac{1.5}{999} = \frac{999e^{-.6030x}}{999}$$

$$.0015 = e^{-.6030x}$$

$$\ln(.0015) = -.6030x$$

$$X = 10.78$$

NEWTON'S LAW OF COOLING

The temperature of an object, T, in surrounding air with temperature T_s will behave according to the formula

$$T(t) = Ae^{kt} + T_s$$

where

- t is time
- A is the difference between the initial temperature of the object and the surroundings
- k is a constant, the continuous rate of cooling of the object

EXAMPLE 5

Using Newton's Law of Cooling

A cheesecake is taken out of the oven with an ideal internal temperature of $165^\circ\mathrm{F}$, and is placed into a $35^\circ\mathrm{F}$ refrigerator. After 10 minutes, the cheesecake has cooled to $150^\circ\mathrm{F}$. If we must wait until the cheesecake has cooled to $70^\circ\mathrm{F}$ before we eat it, how long will we have to wait?

Carbon 14 Dating

$$k = \frac{(\ln \frac{1}{2})}{5730}$$

$$= -.00012$$

EXAMPLE 3

Finding the Age of a Bone

A bone fragment is found that contains 20% of its original carbon-14. To the nearest year, how old is the bone?

$$A = A_0 e^{kt}$$

$$.2 \cancel{A_0} = \cancel{A_0} e^{\frac{\ln(1/2)}{5730} t}$$

$$.2 = e^{\frac{\ln \frac{1}{2}}{5730} t}$$

$$.2 = e^{-.00012t}$$

$$\ln(.2) = -.00012t$$

$$t = \frac{\ln(.2)}{-.00012}$$

$$= 13,412$$

#36) $y = ae^{kt}$

$$.6a = ae^{\frac{\ln \frac{1}{2}}{5730}t}$$

$$.6 = e^{-.00012t}$$

$$\ln(.6) = -.00012t$$

$$t = \frac{\ln(.6)}{-.00012} = 4257 \text{ yrs.}$$

Writing Exp function with base "e"

$f(x) = 5.76(.32)^x$

$e^{\ln .32}$

$5.76e^{(\ln .32)x}$

$5.76e^{-1.13x}$

$g(x) = 2.34(.98)^x$

$2.34\ e^{(\ln .98)x}$

$2.34e^{-.02x}$