NEWTON'S LAW OF COOLING

The temperature of an object, T, in surrounding air with temperature T_s will behave according to the formula

$$T(t) = Ae^{kt} + T_s$$

where

- t is time
- A is the difference between the initial temperature of the object and the surroundings
- k is a constant, the continuous rate of cooling of the object

Find A Find K

EXAMPLE 5

(0,165) (10,150)

$T(t) = Ae^{kt} + T_s$

$T(t) = Ae^{kt} + 35$

Using Newton's Law of Cooling

T_s

A cheesecake is taken out of the oven with an ideal internal temperature of $165^\circ \mathrm{F}$, and is placed into a $35^\circ \mathrm{F}$ refrigerator. After 10 minutes, the cheesecake has cooled to $150^\circ \mathrm{F}$. If we must wait until the cheesecake has cooled to $70^\circ \mathrm{F}$ before we eat it, how long will we have to wait?

$T(t) = 130e^{kt} + 35$

$165 = Ae^{k(0)} + 35$

$165 = A + 35$

$A = 130$

$150 = 130e^{k(10)} + 35$

$-35 \qquad -35$

$\frac{115}{130} = \frac{130e^{10k}}{130}$

$.8846 = e^{10k}$

$K = \frac{\ln .8846}{10}$

$= -.01227$

EXAMPLE 3

Finding the Age of a Bone

A bone fragment is found that contains 20% of its original carbon-14. To the nearest year, how old is the bone?

$A(t) = 130e^{-.01227t} + 35$

$70 = 130e^{-.01227t} + 35$

$35 = 130e^{-.01227t}$

$\frac{35}{130} = e^{-.01227t}$

$t = \frac{\ln\left(\frac{35}{130}\right)}{-.01227}$

107 minutes

$(0, 165)$ $\quad$ $(30, 145)$

$165 = Ae^{k(0)} + 75$

$A = 90$

$A(t) = 90e^{-.00846} + 75$

$$\begin{array}{r} 145 = 90e^{30k} + 75 \\ -75 \qquad\qquad -75 \\ \hline 70 = 90e^{30k} \end{array}$$

$$k = \frac{\ln\left(\frac{7}{9}\right)}{30}$$

$k = -.0084$