

Chapter 3 Transformations

Side lengths stay same length

↗ Angles stay the same measure

A rigid motion is a transformation that preserves length and angle measure. Is the transformation a rigid motion? Explain.

SOLUTION

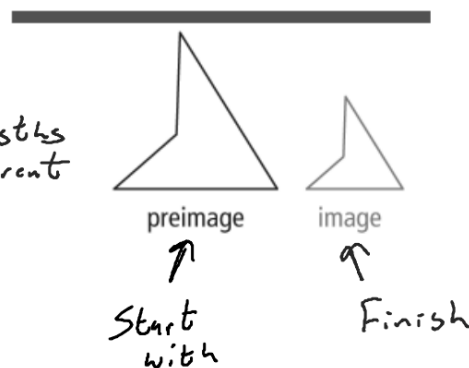
Preimage $\triangle ABC$

Image $\triangle A'B'C'$

A' read "A prime"

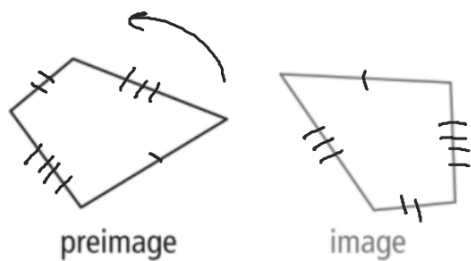
A'' read "A Double Prime"

↳ No side lengths are different



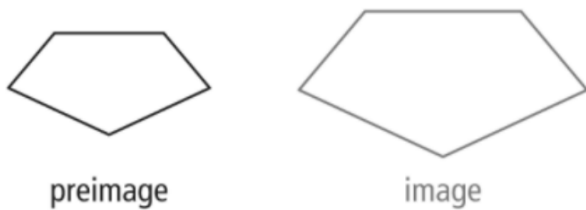
Try It!

1. a. Is the transformation a rigid motion? Explain.



Yes sides same length
Angles same measure.

1. b. Is the transformation a rigid motion? Explain.



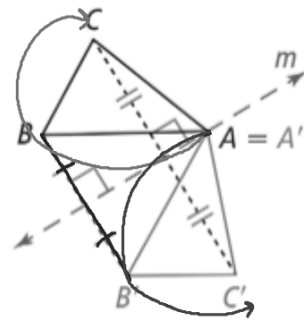
No side lengths are not
the same.

Reflections

A reflection is a transformation that reflects each point in the preimage across a line of reflection.

A reflection has these properties:

- If a point A is on line m , then the point and its image are the same point (that is, $A' = A$).
- If a point B is not on line m , line m is the perpendicular bisector of $\overline{BB'}$.



The reflection of $\triangle ABC$ across line m can be written as $R_m(\triangle ABC) = \triangle A'B'C'$.

A reflection is a rigid motion so length and angle measures are preserved.

Orientation is not preserved

R_m

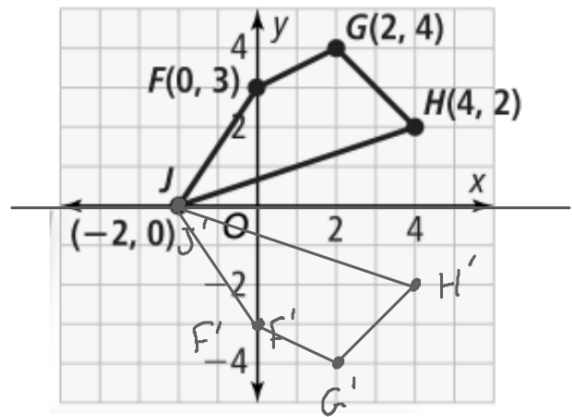
Reflection over line m

Quadrilateral $FGHJ$ has coordinates $F(0, 3)$, $G(2, 4)$, $H(4, 2)$, $J(-2, 0)$.

A. Graph and label $FGHJ$ and $R_{x\text{-axis}}(FGHJ)$. What is a general rule for reflecting a point across the x -axis?

$R_{x\text{-axis}}$ $J(-2, 0)$ $F(0, 3)$
Reflection over
 x -axis $J'(-2, 0)$ $F'(0, -3)$

 $G(2, 4)$ $H(4, 2)$
 $G'(2, -4)$ $H'(4, -2)$



Reflect across x -axis
Change sign on y -value.

$$(x, y) \rightarrow (x, -y)$$

$$R_{x\text{-axis}} \rightarrow (x, -y)$$

Quadrilateral $FGHJ$ has coordinates $F(0, 3)$, $G(2, 4)$, $H(4, 2)$, $J(-2, 0)$.

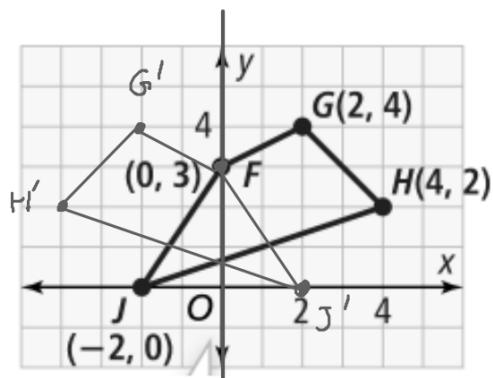
B. Graph and label $FGHJ$ and $R_{y\text{-axis}}(FGHJ)$. What is a general rule for reflecting a point across the y -axis?

$F(0,3)$ $G(2,4)$ $H(4,2)$ $J(-2,0)$
 $F'(0,3)$ $G'(-2,4)$ $H'(-4,2)$ $J'(2,0)$

Reflect over y -axis
Change sign on x -value

$$(x,y) \rightarrow (-x,y)$$

$$R_{y\text{-axis}} \rightarrow (-x,y)$$



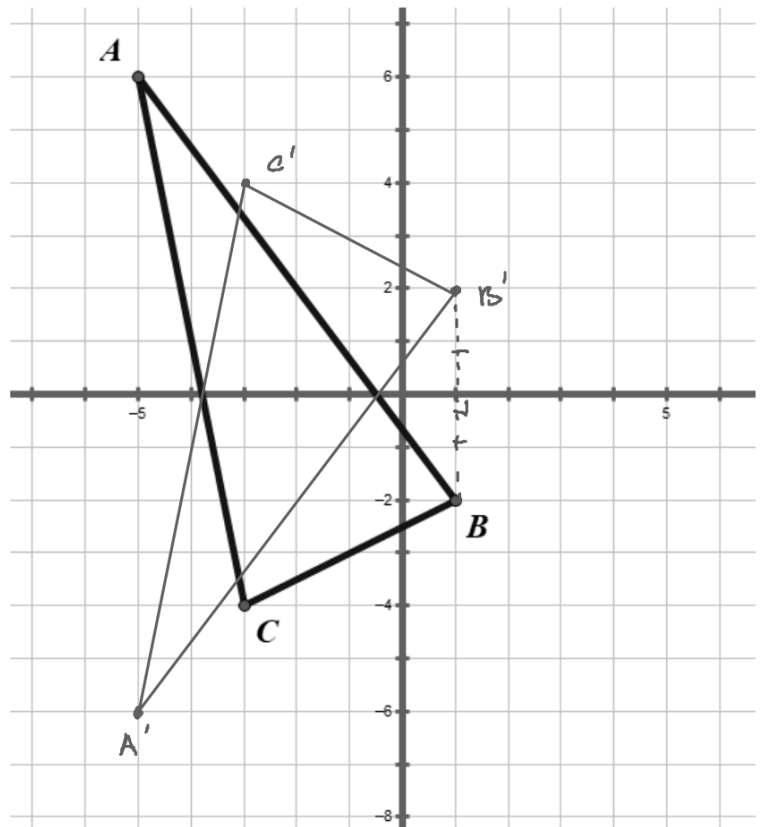
3. Triangle ABC has vertices $A(-5, 6)$, $B(1, -2)$, and $C(-3, -4)$. What are the coordinates of the vertices of $\triangle A'B'C'$ for each reflection?

a. $R_{x\text{-axis}}$ $(x, y) \rightarrow (x, -y)$

$$A'(-5, -6)$$

$$B'(1, 2)$$

$$C'(-3, 4)$$



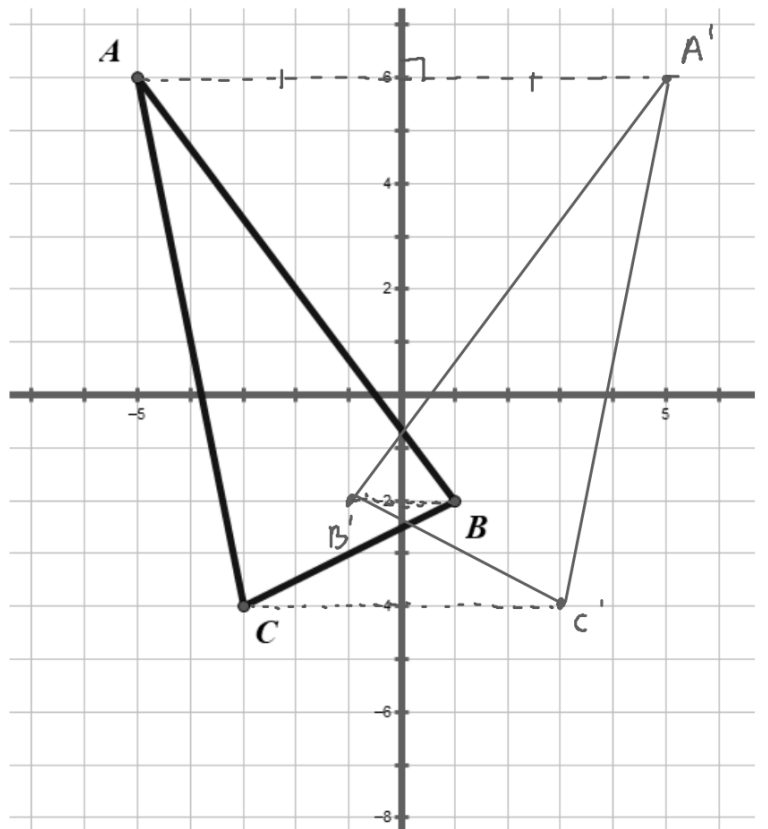
3. Triangle ABC has vertices $A(-5, 6)$, $B(1, -2)$, and $C(-3, -4)$. What are the coordinates of the vertices of $\triangle A'B'C'$ for each reflection?

$R_{y\text{-axis}}$ $(x, y) \rightarrow (-x, y)$

$$A'(5, 6)$$

$$B'(-1, -2)$$

$$C'(3, -4)$$



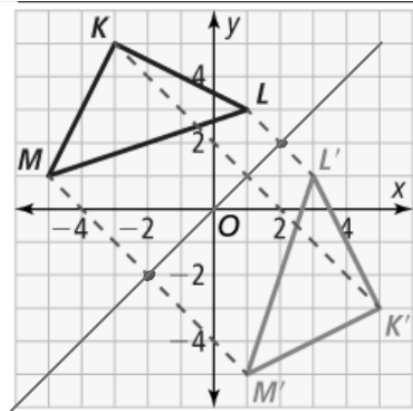
What reflection rule maps $\triangle KLM$ to its image?

Step 1

Write the coordinates of the preimage and the image.

$$\begin{array}{ccc} K(-3, 5) & L(1, 3) & M(-5, 1) \\ K'(5, -3) & L'(3, 1) & M'(1, -5) \end{array}$$

$\begin{matrix} x_1 & y_1 \\ x_2 & y_2 \end{matrix}$



Step 2 Find mid point of $\overline{LL'}$ and $\overline{MM'}$

$$\overline{LL'} = \left(\frac{1+3}{2}, \frac{3+1}{2} \right) = (2, 2)$$

$$\overline{MM'} = \left(\frac{-5+1}{2}, \frac{1+(-5)}{2} \right) = (-2, -2)$$

midpt formula

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$

Step 3 Find the slope using midpts

$$\begin{matrix} x_1 & y_1 \\ (2, 2) & (-2, -2) \end{matrix}$$

$$m = \frac{-2-2}{-2-2} = \frac{-4}{-4} = 1$$

Slope formula

$$m = \frac{y_2-y_1}{x_2-x_1}$$

Step 4 Use point-slope form to find the equation of the line of reflection.

$$m=1 \quad \begin{matrix} (2, 2) \\ x_1, y_1 \end{matrix}$$

$$y-2 = 1(x-2)$$

$$y-2 = x-2$$

$$y = x$$

pt-slope form

$$y-y_1 = m(x-x_1)$$

4. What is a reflection rule that maps each triangle to its image?

a. $C(3, 8)$, $D(5, 12)$, $E(4, 6)$ and
 $C'(-8, -3)$, $D'(-12, -5)$, $E'(-6, -4)$

Step 2 Find midpt

$$\overline{CC'} \quad \overline{EE'}$$
$$\left(\frac{3+(-8)}{2}, \frac{8+(-3)}{2} \right) \quad \left(\frac{4+(-6)}{2}, \frac{6+(-4)}{2} \right)$$
$$(-2.5, 2.5) \quad (-1, 1)$$

Step 3 Slope using midpts

$$m = \frac{1-2.5}{-1-(-2.5)}$$

